

Date
Subject

جواب تین حصوں میں

$$f(z) = w = \frac{1}{z} \rightarrow z = \frac{1}{w} \rightarrow -1 \in \star$$

$$z = \frac{u}{u^2 + v^2} - i \frac{v}{u^2 + v^2} \rightarrow x = \frac{u}{u^2 + v^2},$$

$$y = \frac{-v}{u^2 + v^2}$$

$$x^2 - y^2 = 1 \rightarrow \frac{u^2 - v^2}{(u^2 + v^2)^2} = 1 \rightarrow u^2 - v^2 = (u^2 + v^2)^2$$

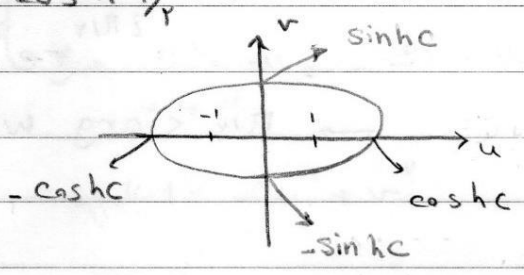
$$\begin{cases} u = \cos x \cosh y \\ v = \sin x \sinh y \end{cases} \quad \left. \begin{array}{l} y = c \\ 1/r \leq c < 1 \end{array} \right\} \quad \frac{u^r}{\cosh^r c} + \frac{v^r}{\sinh^r c} = 1 \quad (b = \varepsilon \in \mathbb{R}^+)$$

$$e = c/a = \frac{\sqrt{a^r - b^r}}{a} = \frac{1}{\cosh c}$$

$$1/r \leq c \leq 1 \rightarrow \cosh 1/r \leq \cosh c \leq \cosh 1$$

$$\cosh 1 \leq \frac{1}{\cosh c} \leq \cosh 1/r$$

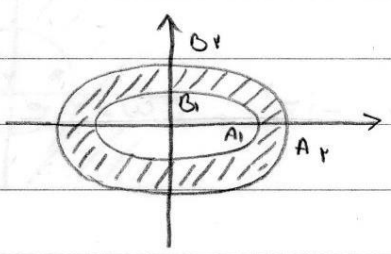
$$\forall c \in [1/r, 1]$$



مربعات $\cosh x$ و $\sinh x$ در هر دو جهت مثبت و منفی

$$A_1 = \begin{vmatrix} \cosh 1/r \\ 0 \end{vmatrix} \quad B_1 = \begin{vmatrix} \sinh 1/r \\ 0 \end{vmatrix}$$

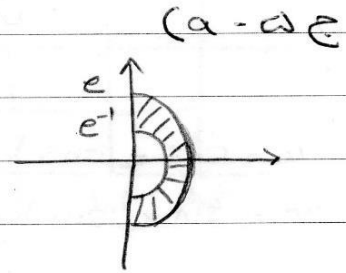
$$A_r = \begin{vmatrix} \cosh 1 \\ 0 \end{vmatrix} \quad B_r = \begin{vmatrix} \sinh 1 \\ 0 \end{vmatrix}$$



$$w = e^x e^{iy} \quad \rho = e^x$$

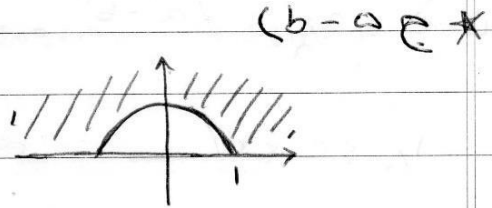
$$-1 < x < 1 \rightarrow e^{-1} < \rho < e$$

$$\text{arg } w = y \quad -\pi/2 < y < \pi/2$$



$$x \geq 0 \rightarrow \rho \geq 1$$

$$0 < y < \pi \rightarrow 0 < \varphi < \pi$$



$$Z^2 = (re^{i\theta})^2 \Rightarrow \rho e^{i\phi} = r^2 e^{i2\theta} \rightarrow \begin{cases} \phi = 2\theta \\ \rho = r^2 \end{cases}$$

$$\therefore Z^2 = \sqrt{z} \cdot \sqrt{z} = 1$$

$$Z^2 = (x+iy)^2 = x^2 + i^2 y^2 + 2ixy = \underbrace{(x^2 - y^2)}_u + i \underbrace{(2xy)}_v$$

a. $y = 1+x$

$$\begin{aligned} \rightarrow u &= x^2 - (1+x)^2 = x^2 - 1 - x^2 - 2x = -(1+2x) \rightarrow u = -1-2x \rightarrow x = \frac{-u-1}{2} \\ \rightarrow v &= 2xy = 2x(1+x) \rightarrow v = -2 \left(\frac{1+u}{2} \right) \left(1 - \frac{(u+1)}{2} \right) \\ \rightarrow v &= -(1+u) \left(\frac{2-u-1}{2} \right) = -\frac{1}{2} (1+u)(1-u) = -\frac{1}{2} (1-u^2) \\ &\rightarrow v = \frac{1}{2} (u^2 - 1) \end{aligned}$$

b. $y^2 = x^2 + 1$

$$\begin{aligned} \rightarrow u &= x^2 - y^2 = x^2 - x^2 - 1 \rightarrow u = -1 \\ \rightarrow v &= 2xy \Rightarrow v^2 = 4x^2 y^2 = 4x^2 (x^2 + 1) \end{aligned} \rightarrow \begin{cases} u = -1 \\ v = 2x \sqrt{1+x^2} \end{cases}$$